2024 秋季本科时间序列

第 6 次作业答案

11 月 24 日

1. (a)

$$\boldsymbol{A}(\mathscr{L}) = \boldsymbol{I} - \boldsymbol{\Phi}\mathscr{L} = \begin{bmatrix} 1-0.4\mathscr{L} & 0.2\mathscr{L} & -\mathscr{L} \\ -0.1\mathscr{L} & 1-0.7\mathscr{L} & -\mathscr{L} \\ 0 & 0 & 1-0.8\mathscr{L} \end{bmatrix}$$

(b)

$$\begin{aligned} \det \boldsymbol{A}(\mathscr{L}) &= (1-0.4\mathscr{L})(1-0.7\mathscr{L})(1-0.8\mathscr{L}) - 0.2\mathscr{L}(-0.1\mathscr{L})(1-0.8\mathscr{L}) \\ &= (1-0.8\mathscr{L})(1-1.1\mathscr{L}+0.3\mathscr{L}^2) \\ &= 1-1.9\mathscr{L}+1.18\mathscr{L}^2-0.24\mathscr{L}^3 \end{aligned}$$

$$\boldsymbol{A}^*(\mathscr{L}) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$\boldsymbol{A}^*(\mathscr{L})$ 是 $\boldsymbol{A}(\mathscr{L})$ 的伴随矩阵，A_{ij} 是代数余子式，可计算得到

$$A_{11} = \det \begin{bmatrix} 1-0.7\mathscr{L} & -\mathscr{L} \\ 0 & 1-0.8\mathscr{L} \end{bmatrix} = 1-1.5\mathscr{L}+0.56\mathscr{L}^2$$

$$A_{12} = -\det \begin{bmatrix} -0.1\mathscr{L} & -\mathscr{L} \\ 0 & 1-0.8\mathscr{L} \end{bmatrix} = 0.1\mathscr{L}-0.08\mathscr{L}^2$$

$$A_{13} = \det \begin{bmatrix} -0.1\mathscr{L} & 1-0.7\mathscr{L} \\ 0 & 0 \end{bmatrix} = 0$$

$$A_{21} = -\det \begin{bmatrix} 0.2\mathscr{L} & -\mathscr{L} \\ 0 & 1-0.8\mathscr{L} \end{bmatrix} = -0.2\mathscr{L} + 0.16\mathscr{L}^2$$

$$A_{22} = \det \begin{bmatrix} 1-0.4\mathscr{L} & -\mathscr{L} \\ 0 & 1-0.8\mathscr{L} \end{bmatrix} = 1 - 1.2\mathscr{L} + 0.32\mathscr{L}^2$$

$$A_{23} = -\det \begin{bmatrix} 1-0.4\mathscr{L} & 0.2\mathscr{L} \\ 0 & 0 \end{bmatrix} = 0$$

$$A_{31} = \det \begin{bmatrix} 0.2\mathscr{L} & -\mathscr{L} \\ 1-0.7\mathscr{L} & -\mathscr{L} \end{bmatrix} = \mathscr{L} - 0.9\mathscr{L}^2$$

$$A_{32} = -\det \begin{bmatrix} 1-0.4\mathscr{L} & -\mathscr{L} \\ -0.1\mathscr{L} & -\mathscr{L} \end{bmatrix} = \mathscr{L} - 0.3\mathscr{L}^2$$

$$A_{33} = \det \begin{bmatrix} 1-0.4\mathscr{L} & 0.2\mathscr{L} \\ -0.1\mathscr{L} & 1-0.7\mathscr{L} \end{bmatrix} = 1 - 1.1\mathscr{L} + 0.3\mathscr{L}^2$$

所以

$$\boldsymbol{A}^*(\mathscr{L}) = \begin{bmatrix} 1-1.5\mathscr{L}+0.56\mathscr{L}^2 & -0.2\mathscr{L}+0.16\mathscr{L}^2 & \mathscr{L}-0.9\mathscr{L}^2 \\ 0.1\mathscr{L}-0.08\mathscr{L}^2 & 1-1.2\mathscr{L}+0.32\mathscr{L}^2 & \mathscr{L}-0.3\mathscr{L}^2 \\ 0 & 0 & 1-1.1\mathscr{L}+0.3\mathscr{L}^2 \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{A}^{-1}(\mathscr{L}) &= \frac{1}{\det \boldsymbol{A}(\mathscr{L})}\boldsymbol{A}^*(\mathscr{L}) \\ &= \frac{1}{1-1.9\mathscr{L}+1.18\mathscr{L}^2-0.24\mathscr{L}^3} \begin{bmatrix} 1-1.5\mathscr{L}+0.56\mathscr{L}^2 & -0.2\mathscr{L}+0.16\mathscr{L}^2 & \mathscr{L}-0.9\mathscr{L}^2 \\ 0.1\mathscr{L}-0.08\mathscr{L}^2 & 1-1.2\mathscr{L}+0.32\mathscr{L}^2 & \mathscr{L}-0.3\mathscr{L}^2 \\ 0 & 0 & 1-1.1\mathscr{L}+0.3\mathscr{L}^2 \end{bmatrix} \end{aligned}$$

(c)

$$
\begin{aligned}
P(z) &= \det \boldsymbol{A}(z) \\
&= 1 - 1.9z + 1.18z^2 - 0.24z^3 \\
&= -0.02(z-2)(4z-5)(3z-5)
\end{aligned}
$$

令 $P(z) = 0$ 得 $z_1 = 2, z_2 = \frac{5}{4}, z_3 = \frac{5}{3}$，均满足 $z_i > 1$，即三个零点均在单位圆外，$\boldsymbol{X}_t$ 满足平稳性

(d) 特征值 λ 满足 $\det(\boldsymbol{\Phi} - \lambda \boldsymbol{I}) = 0$

$$
\begin{aligned}
\det(\boldsymbol{\Phi} - \lambda \boldsymbol{I}) &= \det \begin{bmatrix} 0.4-\lambda & -0.2 & 1 \\ 0.1 & 0.7-\lambda & 1 \\ 0 & 0 & 0.8-\lambda \end{bmatrix} \\
&= (0.4-\lambda)(0.7-\lambda)(0.8-\lambda) + 0.02(0.8-\lambda) \\
&= -\lambda^3 + 1.9\lambda^2 - 1.18\lambda + 0.24 \\
&= 0
\end{aligned}
$$

解得 $\lambda_1 = \frac{1}{2}, \lambda_2 = \frac{4}{5}, \lambda_3 = \frac{3}{5}$，所以 $\lambda_i z_i = 1$，即 z_i 与 λ_i 互为倒数，对于 $i = 1, 2, 3$ 均成立