Monetary Policy According to HANK

Presenter: Zhentao Zou

EMS, Wuhan University

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Motivation

- Changes in interest rates influence household consumption through both *direct* and *indirect* effects.
- Monetary policy in RANK models works almost exclusively through direct effects.
- Macroeconometric analysis of aggregate time-series data finds a *small* sensitivity of consumption to changes in the interest rate after controlling for income.
- Revisit the transmission mechanism from monetary policy to household consumption in a HANK model with two features:
  1. uninsurable income shocks;
  2. assets with different degrees of liquidity.

Benchmark: RANK

- Goal: *introduce decomposition of C response to r change*.
- A representative household has CRRA utility from consumption C_t with parameter γ, and discounts the future at rate ρ.
- The household can borrow and save at rate r_t.
- A representative firm produces output using only labor, according to the production function $Y = N$.
- Both wage and final goods price are perfectly rigid and normalized to 1.
- The household commits to supplying any amount of labor demanded at the prevailing wage so that its labor income equals Y_t in every instant.
- In equilibrium, the goods market clears $C_t = Y_t$.

Overall Effect of Monetary Policy

- Household optimization implies the Euler equation:

$$\frac{dC_t}{C_t} = \frac{r_t - \rho}{\gamma} dt. \tag{1}$$

- We assume that the economy returns to its steady-state level in the long-run, $C_t \to \overline{C}$ as $t \to \infty$.

- It therefore follows that

$$logC_0 = log\overline{C} - \frac{1}{\gamma}\int_0^{\infty}(r_t - \rho)dt. \tag{2}$$

- The overall effect on initial consumption

$$dlogC_0 = -\frac{1}{\gamma}\int_0^{\infty} dr_t dt. \tag{3}$$

Decomposition into Direct and Indirect Effects

- Assume that initially $r_t = \rho$ and $Y_t = \overline{Y}$ for all t.
- We use a perturbation argument around the steady state:

$$dlogC_0 = \underbrace{\int_0^\infty \frac{\partial logC_0}{\partial r_t} dr_t dt}_{\text{direct effect}} + \underbrace{\int_0^\infty \frac{\partial logC_0}{\partial logY_t} dlogY_t dt}_{\text{indirect effect}}. \tag{4}$$

- Since

$$\frac{\partial logC_0}{\partial r_t} = -\frac{e^{-\rho t}}{\gamma}, \quad \frac{\partial logC_0}{\partial logY_t} = \rho e^{-\rho t}, \tag{5}$$

we obtain

$$dlogC_0 = \underbrace{-\frac{1}{\gamma}\int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct effect}} \underbrace{-\frac{\rho}{\gamma}\int_0^\infty e^{-\rho t}\int_t^\infty dr_s ds dt}_{\text{indirect effect}}. \tag{6}$$

Special Case

- Clean and intuitive formulae can be obtained for the special case

$$r_t = \rho + e^{-\eta t}(r_0 - \rho), \tag{7}$$

where the interest rate *unexpectedly* jumps at $t = 0$ and then mean reverts at rate $\eta > 0$.

- The decomposition becomes

$$\frac{dlogC_0}{dr_0} = \underbrace{-\frac{1}{\gamma(\rho+\eta)}}_{\text{direct effect}} \underbrace{-\frac{\rho}{\gamma\eta(\rho+\eta)}}_{\text{indirect effect}} = \underbrace{-\frac{1}{\gamma\eta}}_{\text{overall effect}} . \tag{8}$$

- Hence we have

$$\frac{indirect}{overall} = \frac{\rho}{\rho+\eta}, \tag{9}$$

which is less than 10% for reasonable parameterizations.

HANK Model: Households

- The economy is populated by a continuum of households indexed by their holdings of liquid assets b, illiquid assets a, and their idiosyncratic labor productivity z.
- We assume the logarithm of labor productivity $x_{it} = logz_{it}$ follows a jump-drift process
$$dx_{it} = -\beta x_{it} dt + dJ_{it}, \tag{10}$$
where jumps arrive at a Poisson rate λ.
- Conditional on a jump, a new log productivity x'_{it} is drawn from a normal distribution with mean zero and variance σ^2.
- At time t, the state of the economy is the joint distribution $\mu_t(da, db, dx)$.

Household's Problem

- Preferences are time-separable as

$$\mathbb{E}\int_0^\infty e^{-\rho t}u(c_t, l_t)dt, \tag{11}$$

where the expectation is taken over realizations of idiosyncratic productivity shocks.

- A households asset holdings evolve according to

$$da_t = (r_t^a a_t + d_t)dt,\ a_t \geq 0. \tag{12}$$

$$db_t = [e^{x_t} w_t l_t + r_t^b b_t - d_t - \phi(d_t, a_t) - c_t]dt,\ b_t \geq -\underline{b}; \tag{13}$$

- Households take as given equilibrium paths for w_t, r_t^b and r_t^a.

HJB and KF Equations

- The stationary version of households' HJB equation is then given by

$$\begin{aligned}\rho V(a,b,x) &= \max_{c,l,d} u(c,l) + (r^a a + d)V_a \\ &+[e^x wl + r^b b - d - \phi(d,a) - c]V_b - \beta x V_x \\ &+\lambda \int_{-\infty}^{\infty} [V(a,b,x') - V(a,b,x)]\psi(x')dx', \qquad (14)\end{aligned}$$

where ψ is the density of a normal distribution with variance σ^2.

- The stationary density satisfies the Kolmogorov forward equation

$$\begin{aligned}0 &= \partial_a[s^a(a,b,x)g(a,b,x)] + \partial_b[s^b(a,b,x)g(a,b,x)] \\ &+\partial_x[-\beta x g(a,b,x)] + \lambda g(a,b,x) - \lambda\psi(x)\int_{-\infty}^{\infty} g(a,b,x)dx, \qquad (15)\end{aligned}$$

where $s^a(a,b,x)$ and $s^b(a,b,x)$ are the optimal liquid and illiquid asset saving policy functions, respectively.

Final-Goods Producers

- A competitive representative final-good producer aggregates a continuum of intermediate inputs:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon - 1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon - 1}}, \tag{16}$$

  where $\epsilon > 0$ is the elasticity of substitution across goods.

- Cost minimization implies that demand for intermediate good j is

$$y_{jt}(p_{jt}) = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t, \text{ where } P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{17}$$

Intermediate Goods Producers

- Each intermediate good j is produced by a monopolistically competitive producer

$$y_{jt} = k_{jt}^{\alpha} n_{jt}^{1-\alpha}. \tag{18}$$

- Intermediate producers rent capital at rate r_t^k in a competitive capital market and hire labor at wage w_t in a competitive labor market.

- Cost minimization implies that the marginal cost is common across all producers and given by

$$m_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{19}$$

Price Adjustment: Rotemberg (1982)

- Each intermediate producer chooses $\hat{\pi}_t = \frac{dp_t/dt}{p_t}$ to maximize

$$\int_0^{\infty} e^{-\int_0^t r_s^a ds} \left[\left(\frac{p_t}{P_t} - m_t \right) \left(\frac{p_t}{P_t} \right)^{-\epsilon} Y_t - \frac{\theta}{2} \hat{\pi}_t^2 Y_t \right] dt. \qquad (20)$$

- The choice of r_t^a for the rate at which firms discount future profits is justified by a no-arbitrage condition.

- A continuous-time formulation of the problem yield the New Keynesian Phillips curve *without* the need for log-linearization:

$$d\pi_t = -\pi_t \left(\frac{dY_t}{Y_t} - r_t^a dt \right) - \frac{\epsilon}{\theta} \left(m_t - \frac{\epsilon - 1}{\epsilon} \right) dt. \qquad (21)$$

Composition of Illiquid Wealth

- Illiquid savings can be invested in two assets: (i) capital k_t, and (ii) equity shares of the aggregate portfolio of intermediate firms.

- This equity represents a claim on the entire future stream of monopoly profits net of price adjustment costs:

$$\Pi_t = (1 - m_t) Y_t - \frac{\theta}{2} \pi_t^2 Y_t. \tag{22}$$

- Let q_t denote the share price:

$$q_t = \int_t^\infty e^{-\int_t^\tau r_s^a ds} \Pi_\tau d\tau. \tag{23}$$

- No-arbitrage condition implies that

$$\frac{\Pi_t dt + dq_t}{q_t} = (r_t^k - \delta) dt = r_t^a dt. \tag{24}$$

Monetary Authority

- The monetary authority sets the nominal interest rate on liquid assets according to a Taylor rule:

$$i_t = \bar{r}^b + \psi\pi_t + z_t, \tag{25}$$

  where $\psi > 1$ and $z_t = 0$ in steady state.

- Our main experiment studies the economys adjustment after an unexpected temporary monetary shock z_t.

- The real return on the liquid asset is determined by the Fisher equation

$$r_t^b = i_t - \pi_t. \tag{26}$$

Equilibrium

- The liquid asset market clears:

$$\int b d\mu_t = 0. \tag{27}$$

- The illiquid asset market clears:

$$\int a d\mu_t = K_t + q_t. \tag{28}$$

- The labor market clears:

$$\int e^x l_t(a, b, x) d\mu_t = N_t. \tag{29}$$

- The good market clears:

$$Y_t = C_t + I_t + \frac{\theta}{2}\pi_t^2 Y_t + \phi_t + \kappa \int max(-b, 0) d\mu_t. \tag{30}$$

Monetary Transmission in HANK

We decompose the consumption response at $t = 0$ as

$$dlogC_0 = \underbrace{\int_0^\infty \frac{\partial logC_0}{\partial r_t^b} dr_t^b dt}_{\text{direct effect}} + \underbrace{\int_0^\infty \left(\frac{\partial logC_0}{\partial w_t} dw_t + \frac{\partial logC_0}{\partial r_t^a} dr_t^a \right) dt}_{\text{indirect effect}}, \tag{31}$$

where

$$\frac{\partial logC_0}{\partial r_t^b} = \int \frac{\partial logc_0(a, b, x; r_t^b, \bar{r}^a, \overline{w})}{\partial r_t^b} d\mu_0^{r^b}, \tag{32}$$

$$\frac{\partial logC_0}{\partial w_t} = \int \frac{\partial logc_0(a, b, x; \bar{r}^b, \bar{r}^a, w_t)}{\partial w_t} d\mu_0^{w}, \tag{33}$$

$$\frac{\partial logC_0}{\partial r_t^a} = \int \frac{\partial logc_0(a, b, x; \overline{r^b}, r_t^a, \overline{w})}{\partial r_t^a} d\mu_0^{r^a}. \tag{34}$$

Main Finding

- In stark contrast to RANK economies, the direct effects of interest rate shocks in our HANK model are always small, while the indirect effects can be substantial.
- Uninsurable risk, combined with the coexistence of liquid and illiquid assets in financial portfolios, leads to the presence of a sizable fraction of poor and wealthy hand-to-mouth households, as in the data.
- These households are highly sensitive to labor income shocks but are not responsive to interest rate changes.